Coláiste Mhuire Gan Smál
– Ollscoil Luimnigh –

Mary Immaculate College
– University of Limerick –

Mid-Semester Assessment paper

Module Code: MH 4728
Module Title: Abstract Algebra
Lecturer: Dr. B. Kreussler
External Examiner: Prof. D. Lawson
Semester: Spring 2012
Duration of exam: 45 minutes
Percentage of total marks: 25%
Authorised Materials: Calculator

Instructions to candidates: Answer **one** of the following **two** questions.
Please show your work.

Question 1:

10 marks (a) Find all integers x which satisfy the following congruences simultaneously.

$$\begin{aligned} 7\,x &\equiv 15 \mod 17 \\ 13\,x &\equiv 11 \mod 29 \\ 18\,x &\equiv 55 \mod 121 \end{aligned}$$

9 marks (b) Find the smallest positive integers n which satisfies $n^{43} \equiv 142 \mod 189$.

6 marks (c) Let x be a given real number. Use induction to prove for all $n \geq 1$

$$(1-x)^2 \sum_{k=1}^{n} kx^k = x - (n+1)x^{n+1} + nx^{n+2}\,.$$

Question 2:

10 marks (a) For each of the elements $[321], [322], [323], [324], [325]$ and $[326]$ in $\mathbb{Z}/759\,\mathbb{Z}$

- ▶ decide whether or not it has a multiplicative inverse;
- ▶ if it has a multiplicative inverse, compute it.

9 marks (b) Find all integers x which satisfy the following congruences simultaneously.

$$\begin{aligned} x^{173} &\equiv 4 \mod 945 \\ 29\,x &\equiv 1 \mod 64 \end{aligned}$$

6 marks (c) Find the smallest positive integer n such that $90^{2000} + 91^{1999} \equiv n \mod 375$.