Euclidean and Non-Euclidean Geometry – Exercise Sheet 4

Problem 4.1 Draw a triangle ABC. Construct a point P such that $AP = \frac{5}{3}AB$. Write down a rough description of your construction.

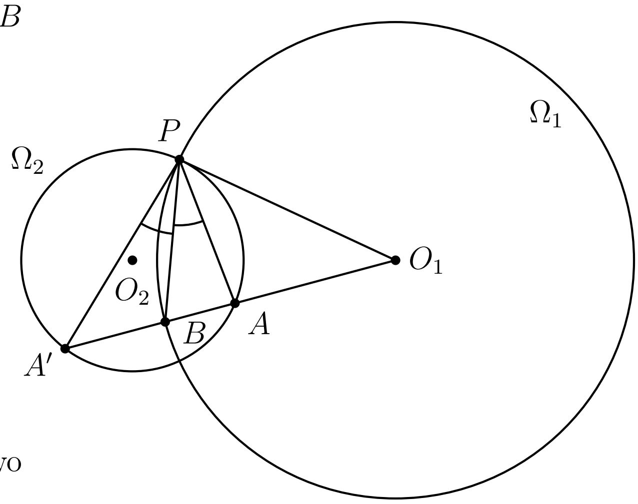
Problem 4.2 Triangle ABC is placed in a co-ordinate system such that A is the origin and B is on the positive x-axis. Given that $a = 28$, $b = 45$ and $c = 53$, determine the **equation of the circumcircle** of the triangle ABC.

HINT: First determine the angle γ.

Problem 4.3 Let Ω_1 and Ω_2 be perpendicular circles with centres O_1 and O_2, respectively. Let P be one of the intersection points of Ω_1 and Ω_2 and B a point on Ω_1 which is inside Ω_2. The points A and A' are the points where the line BO_1 intersects the circle Ω_2.

Using the steps below, **prove** that PB is the **angle bisector** of $\angle APA'$.

1. $\angle BPO_1 = \angle PBO_1$
2. $\angle BPO_1 = \angle APO_1 + \angle BPA$
3. $\angle PBO_1 = \angle PA'B + \angle BPA'$
4. $\angle APO_1 = \angle PA'B$
5. $\angle BPA' = \angle BPA$



At which step did you use that the two circles are perpendicular?

Problem 4.4 (Dutta's Construction of Inversion)
Mark three non-collinear points A, O_1, P such that $O_1P > O_1A$ and draw the circle Ω_1 of radius O_1P that is centred at O_1.

(a) Draw ray O_1A and label B its intersection point with Ω_1.

Draw circle centre B of radius BA and draw circle centre P of radius PA. Label C their new intersection point.

Let A' be the intersection point of O_1A and PC.

(b) Use exercise 4.4 to justify that inversion in Ω_1 maps A to A'.

Problem 4.5 (Fractions and Functions)

(a) Without using a calculator, write as a single fraction in shortest terms.

(i) $1 - \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} - \frac{1}{6}$ (ii) $\dfrac{\frac{3a}{bc} - \frac{2b}{b-a}}{6ab}$

(b) Simplify!
(i) $(3x-2)^2$ (ii) $\ln(25(x-1)^3)$