Introduction to Geometry – Exercise Sheet 1

Problem 1.1 For any positive integers u and v that satisfy u > v we define

 $a = u^2 - v^2$, b = 2uv and $c = u^2 + v^2$.

Prove that (a, b, c) is a Pythagorean Triple.

Problem 1.2 With the aid of the previous problem, find at least one Pythagorean Triple (a, b, c) for which:

- (i) c = 29
- (ii) c = 37
- (iii) c = 41
- (iv) c = 53
- (v) c = 61

Find at least two Pythagorean Triples (a, b, c) for which:

- (vi) c = 65
- (vii) c = 85

Can you find two more Pythagorean Triples with c = 65?

Problem 1.3 Explain what it means to divide a line segment in the golden ratio.

Problem 1.4 Given a line segment AB. Construct with ruler and compass a point F on AB such that

$$\frac{AB}{BF} = \frac{BF}{AF}.$$

Problem 1.5 Given a line segment AB of length a. Construct with ruler and compass a line segment of length 15a. Try to use as few steps as possible.

Problem 1.6 Given a line segment AB of length a. Construct with ruler and compass a line segment of length:

- (i) $\sqrt{13} a$
- (ii) $\sqrt{14} a$
- (iii) $\sqrt{15} a$