## Introduction to Geometry - Exercise Sheet 1

Problem 1.1 For any positive integers $u$ and $v$ that satisfy $u>v$ we define

$$
a=u^{2}-v^{2}, \quad b=2 u v \quad \text { and } \quad c=u^{2}+v^{2} .
$$

Prove that $(a, b, c)$ is a Pythagorean Triple.
Problem 1.2 With the aid of the previous problem, find at least one Pythagorean Triple ( $a, b, c$ ) for which:
(i) $c=29$
(ii) $c=37$
(iii) $c=41$
(iv) $c=53$
(v) $c=61$

Find at least two Pythagorean Triples ( $a, b, c$ ) for which:
(vi) $c=65$
(vii) $c=85$

Can you find two more Pythagorean Triples with $c=65$ ?
Problem 1.3 Explain what it means to divide a line segment in the golden ratio.
Problem 1.4 Given a line segment $A B$. Construct with ruler and compass a point $F$ on $A B$ such that

$$
\frac{A B}{B F}=\frac{B F}{A F}
$$

Problem 1.5 Given a line segment $A B$ of length $a$. Construct with ruler and compass a line segment of length $15 a$. Try to use as few steps as possible.

Problem 1.6 Given a line segment $A B$ of length $a$. Construct with ruler and compass a line segment of length:
(i) $\sqrt{13} a$
(ii) $\sqrt{14} a$
(iii) $\sqrt{15} a$

